Unitary Symmetry and Proton-Antiproton Interactions*

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An investigation has been made of the conclusions which can be drawn from the unitary symmetry theory of strong interactions about the class of reactions $p+\bar{p} \rightarrow A+\bar{B}$, where A represents a member of the meson or baryon octet, or the baryon resonance decuplet, and $\bar{B}$ is a member of the corresponding antiparticle multiplet. We find that in the case of baryon or meson production, the only consequences are a set of inequalities on cross sections which are both weak and experimentally inaccessible. In the case of production of tenfold resonances, we obtain one equality among cross sections, which, however, involves the experimentally difficult reaction in which the final state is an $N^{*-}\bar{N}^{*+}$ pair. In addition there are several inequalities. One set of these, namely $\sqrt{\sigma_Z} \geq |\sqrt{\sigma_Y} - 2\sqrt{\sigma_\Xi}|$, $\sqrt{\sigma_Y} \geq |\sqrt{\sigma_Z} - 2\sqrt{\sigma_\Xi}|$, and $\sqrt{\sigma_\Xi} \geq \frac{1}{2}|\sqrt{\sigma_Z} - \sqrt{\sigma_Y}|$ where σ_Y, σ_Ξ, and σ_Z represent the cross sections (differential or total) for the production of $Y_1^{*-}\bar{Y}_1^{*+}$, $\Xi^{*-}\bar{\Xi}^{*+}$, and $Z^-\bar{Z}^+$ pairs, respectively, gives hope of being both relatively restrictive and experimentally feasible. In particular, it gives limits on the production cross section for the as yet undiscovered strangeness minus three Z^- particle. These relations are expected to hold to the extent that the incident energy is large enough that the mass splitting within the unitary multiplets can be ignored.

THE unitary symmetry theory of strong interactions[1,2] yields rather strikingly accurate predictions about the masses of the heavy particles and resonances.[3] In an attempt to find further experimental tests of the theory, an investigation has been undertaken of its predictions for the class of reactions

$$p+\bar{p} \rightarrow A+\bar{B}, \tag{1}$$

where A represents a member of the meson or baryon unitary octet, or the baryon resonance decuplet,[3] and $\bar{B}$ is a member of the corresponding antiparticle multiplet.

The analysis of the consequences in Reaction (1) of invariance under the group SU_3 may be carried out in a manner which is exactly analogous to the familiar analysis of the consequences of invariance under isotopic spin rotations in, for example, pion-nucleon scattering. In the latter case, the assumption of the invariance of the Hamiltonian under isospin rotations allows one to conclude that the scattering amplitude is zero unless the initial and final states transform according to the same irreducible representation of the isotopic spin rotation group, i.e., have the same total isotopic spin. Moreover, if one considers two groups of initial and final states which do transform by the same irreducible representation, then the scattering amplitude is the same for each pair of corresponding states, that is, it does not depend on the z component of the isotopic spin. Hence, one concludes that pion-nucleon scattering can be described by just two independent amplitudes, one for $I=\frac{3}{2}$ and one for $I=\frac{1}{2}$.

Now let us turn to the analysis of Reaction (1), and consider first the case where A and $\bar{B}$ represent a baryon-antibaryon pair. If we impose only isospin and charge conjugation invariance, then the possible reactions (1) are governed by eight independent amplitudes, namely, the amplitudes for the production of $N\bar{N}$, $\Xi\bar{\Xi}$, or $\Sigma\bar{\Sigma}$ pairs in states of total isotopic spin zero or one, for the production of a $\Lambda\bar{\Lambda}$ pair, and for the production of a $\Sigma^0\bar{\Lambda}$ or $\bar{\Sigma}^0\Lambda$ pair. (The effect of charge conjugation invariance is to require the equality of the amplitudes for the latter pair of processes.) Now let us impose invariance under the larger group SU_3. The baryons and antibaryons are each presumed to transform according to an eight-dimensional irreducible representation of SU_3, so that baryon-antibaryon pairs transform according to the reducible 64-dimensional direct product representation. These 64 states can be decomposed into a set of states transforming according to a 27-dimensional irreducible representation, two sets transforming according to two different ten-dimensional representations, two sets which each transform according to the same eight-dimensional irreducible representation, and a single state which transforms according to a one-dimensional irreducible representation.[1] (In terms of our analogy, this corresponds to the decomposition of the set of pion-nucleon states into those linear combinations having total isotopic spin $\frac{3}{2}$ and $\frac{1}{2}$, respectively.) Initial states transforming according to one of the irreducible representations that appear only once in the decomposition can go only into the corresponding final state, the amplitude being the same for all pairs of states belonging to that irreducible representation. However, an initial state belonging to either of the sets which transform according to the eight-dimensional representation can go to either of the two corresponding final states. One thus has eight independent amplitudes. Again, however, one condition (which turns out to be the equality of the amplitudes for the two ten-dimensional representations) must be imposed to ensure charge conjugation invariance. Hence, the assumption of unitary symmetry implies that the eight amplitudes which we had previously may each be expressed as a linear combination of seven other basic amplitudes; that is to say, unitary symmetry

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[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).
[2] Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).
[3] S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963).

implies one condition on the eight amplitudes allowed by isotopic spin conservation alone.

Actually, of course, to be sure that there are no additional restrictions implied by SU_3, one must be sure that the state $p\bar{p}$ actually contains components which transform according to each of the irreducible representations contained in the direct product; if this were not so, some of the seven amplitudes would not contribute to Reaction (1). In order to do this, one must decompose the state $p\bar{p}$ into its components which transform irreducibly; that is to say, one needs the analog for the group SU_3 of the familiar Clebsch-Gordan coefficients which are needed to decompose pion-nucleon states into components having a definite isotopic spin. The required decomposition is given by Glashow and Sakurai,[4] if one simply replaces in their formulas the mesons by the antibaryons having the same isotopic spin and hypercharge quantum numbers. From their results, one sees that the state $p\bar{p}$ does contain all of the possible irreducible representations, so that there is indeed only one extra condition on the eight amplitudes implied by unitary symmetry.

To see what this restriction is, it is convenient to make use of the concept of U spin introduced by Meshkov, Levinson, and Lipkin.[5] Those authors note that, in addition to the isospin rotation group, there are two other subgroups of SU_3 which are isomorphic to the rotation group, one of which they call the U-spin group. Hence, unitary symmetry not only implies the conservation of the total isotopic spin and its z component but of two other quantities which we may call the total U spin and the z component of U spin, and designate by U and U_z. [As usual, of course, U has the significance that $U(U+1)$ is the eigenvalue of the square of the total U spin.] The values of U and U_z for the various baryons, as well as for the mesons and baryon resonances, are summarized in Table I. Since the algebraic properties of the U-spin group are the same as those of the rotation group, U spin adds by the usual rules for adding angular momentum. We note from Table I that the proton has $U=\frac{1}{2}$, $U_z=\frac{1}{2}$ and the antiproton $U=\frac{1}{2}$, $U_z=-\frac{1}{2}$. Hence, the initial state in Reaction (1) can be written as

$$|p\bar{p}\rangle=(|1\rangle+|0\rangle)/\sqrt{2}\,, \tag{2}$$

where $|1\rangle$ and $|0\rangle$ indicate states of total U spin, 1 and 0, respectively. Hence, the amplitudes for the production of the three final states $n\bar{n}$, $\Xi^0\bar{\Xi}^0$, and $\frac{1}{2}(\sqrt{3}\Sigma^0-\Lambda)\times\frac{1}{2}(\sqrt{3}\bar{\Sigma}^0-\bar{\Lambda})$ must each be linear combinations of two basic amplitudes for $U=1$ and $U=0$; the coefficients can, of course, be worked out simply in terms of the appropriate Clebsch-Gordan coefficients. Doing this, one obtains the relation

$$(n\bar{n})+(\Xi^0\bar{\Xi}^0)=\tfrac{3}{2}(\Sigma^0\bar{\Sigma}^0)-\sqrt{3}(\Sigma^0\bar{\Lambda})+\tfrac{1}{2}(\Lambda\bar{\Lambda}), \tag{3}$$

[4] S. L. Glashow and J. J. Sakurai, Nuovo Cimento **25**, 337 (1962).

[5] S. Meshkov, C. A. Levinson, and H. J. Lipkin, Phys. Rev. Letters **10**, 361 (1963).

TABLE I. The quantum numbers U and U_z for the baryons, mesons, and baryon resonance decuplet. The gaps in the table separate different U-spin multiplets. The antiparticles of the baryons and resonances have the same value of U and the negative value of U_z, and are grouped in the same way in U-spin multiplets.

Baryons	U	U_z	Mesons	U	U_z	Resonances	U	U_z
n	1	1	K^0	1	1	N^{*-}	$\frac{3}{2}$	$\frac{3}{2}$
$\frac{1}{2}(\sqrt{3}\Sigma^0-\Lambda)$	1	0	$\frac{1}{2}(\sqrt{3}\pi^0-\eta)$	1	0	Y_1^{*-}	$\frac{3}{2}$	$\frac{1}{2}$
Ξ^0	1	-1	$\bar{K}^0$	1	-1	Ξ^{*-}	$\frac{3}{2}$	$-\frac{1}{2}$
						Z^-	$\frac{3}{2}$	$-\frac{3}{2}$
p	$\frac{1}{2}$	$\frac{1}{2}$	K^+	$\frac{1}{2}$	$\frac{1}{2}$			
Σ^+	$\frac{1}{2}$	$-\frac{1}{2}$	π^+	$\frac{1}{2}$	$-\frac{1}{2}$	N^{*0}	1	1
						Y_1^{*0}	1	0
						Ξ^{*0}	1	-1
Σ^-	$\frac{1}{2}$	$\frac{1}{2}$	π^-	$\frac{1}{2}$	$\frac{1}{2}$			
Ξ^-	$\frac{1}{2}$	$-\frac{1}{2}$	K^-	$\frac{1}{2}$	$-\frac{1}{2}$	N^{*+}	$\frac{1}{2}$	$\frac{1}{2}$
						Y_1^{*+}	$\frac{1}{2}$	$-\frac{1}{2}$
$\frac{1}{2}(\Sigma^0+\sqrt{3}\Lambda)$	0	0	$\frac{1}{2}(\pi^0+\sqrt{3}\eta)$	0	0	N^{*++}	0	0

where we use the notation $(A\bar{B})$ to indicate the amplitude for the production of particles A and $\bar{B}$ in Reaction (1). Eq. (3) yields a series of "pentagonal" inequalities on cross sections, for example,

$$[\sigma(n\bar{n})]^{1/2}\leq[\sigma(\Xi^0\bar{\Xi}^0)]^{1/2}+\tfrac{3}{2}[\sigma(\Sigma^0\bar{\Sigma}^0)]^{1/2}+\sqrt{3}\{[\sigma(\Sigma^0\bar{\Lambda})]^{1/2}+[\sigma(\bar{\Sigma}^0\Lambda)]^{1/2}\}/2+\tfrac{1}{4}[\sigma(\Lambda\bar{\Lambda})]^{1/2} \tag{4}$$

similar to the "triangle" inequalities familiar from the isotopic spin analysis of pion-nucleon scattering. In (4) we use the notation $\sigma(A\bar{B})$ to represent the cross section for the production of A and $\bar{B}$. The relations of the type (4) hold for the differential cross sections at any angle, as well as for the total cross section. Since we have ignored all mass differences, we must, of course, confine ourselves to incident energies in Reaction (1) which are large compared to the mass splittings within the SU_3 multiplets.

As we have seen, unitary symmetry implies only one condition on the eight amplitudes for the production of baryon-antibaryon pairs in (1). Therefore, no further additional conclusions, beyond Eq. (3), can be drawn from the hypothesis of unitary symmetry. Unfortunately, the inequalities of the type (4) which are thus implied are very uninteresting. In the first place, several of the reactions involved in (4) are difficult or impossible to observe experimentally. Moreover, the more terms there are in such inequalities, unless some of the coefficients become very small, the weaker are the conditions implied. Eq. (4) would be satisfied by any model which yielded only roughly equal cross sections for the various inelastic channels involved. Hence, unitary symmetry alone does not make useful predictions about (1) in the case of final states containing baryon-antibaryon pairs.

The situation is much the same in the case of the production of a meson pair in (1), since the mesons also transform according to the eight-dimensional representation of SU_3. Making use of the U-spin assignments for the mesons given in Table I, one can derive in the same way as before the following relation which is exactly analogous to Eq. (3):

$$(K^0\bar{K}^0)+(\bar{K}^0K^0)=\tfrac{3}{2}(\pi^0\pi^0)-\sqrt{3}(\pi^0\eta)+\tfrac{1}{2}(\eta\eta). \quad (5)$$

In (5), the amplitudes $(K^0\bar{K}^0)$ and $(\bar{K}^0K^0)$ are distinguished by the interchange of the momenta of the K^0 and $\bar{K}^0$. In writing (5), we have used the fact that charge-conjugation invariance implies $(\pi\eta)=(\eta\pi)$. As before, (5) yields a series of inequalities which are quite uninteresting because they are very unrestrictive.

We now turn to the production of the baryon resonances belonging to the tenfold representation of SU_3. This has already been observed in the case of pairs involving N^{*++}, Y_1^{*-}, and perhaps Y_1^{*+}.[6,7] To see what restrictions are imposed by unitary symmetry, we first note that, if only isotopic spin conservation is imposed, there are seven independent amplitudes for production of resonance-antiresonance pairs in (1). The final state in (1) now transforms according to a 100-dimensional representation of SU_3 obtained as the direct product of the two ten-dimensional representations. If one decomposes this into its irreducible components,[8] one finds that it contains a 64-dimensional representation in addition to the 27-dimensional, eight-dimensional and one-dimensional representations which we have already encountered. Since the initial state, as we have seen, does not contain any components transforming according to the 64-dimensional representation, but does contain two different components, each of which transforms separately according to the eight-dimensional representation, there are four independent amplitudes if unitary symmetry is imposed; hence, unitary symmetry implies three conditions on the seven independent amplitudes which we had previously.

To obtain these restrictions, it is again convenient to make use of the U-spin concept. We recall that the initial state contains U spin 0 and 1. From Table I we can then easily see that there are only two amplitudes, for $U=0$ and 1, for the four reactions producing N^{*-}, Y_1^{*-}, Ξ^{*-}, and Z^- together with their respective antiparticles. If we make use of the ordinary Clebsch-Gordan coefficients to find the $U=0$ and 1 components of the resonance-antiresonance states, we may obtain first of all the following relation:

$$(Z^-\bar{Z}^+)=-(Y_1^{*-}\bar{Y}_1^{*+})-2(\Xi^{*-}\bar{\Xi}^{*+}). \quad (6)$$

Hence, the following inequalities hold true for the cross sections

$$\sqrt{\sigma_Z}\geq|\sqrt{\sigma_Y}-2\sqrt{\sigma_\Xi}|,$$
$$\sqrt{\sigma_Y}\geq|\sqrt{\sigma_Z}-2\sqrt{\sigma_\Xi}|,$$
$$\sqrt{\sigma_\Xi}\geq\tfrac{1}{2}|\sqrt{\sigma_Z}-\sqrt{\sigma_Y}|, \quad (7)$$

where we use the shorthand notation σ_Y, σ_Ξ, and σ_Z for the cross sections for the production of Y_1^{*-}–$\bar{Y}_1^{*+}$, Ξ^{*-}–$\bar{\Xi}^{*+}$, and Z^-–$\bar{Z}^+$ pairs, respectively. Of particular interest is the fact that the first of the relations (7) sets a lower bound on the cross section for the production of the so far unobserved Z particle of strangeness minus three and predicted mass 1676 MeV, together with its antiparticle, in terms of the cross sections for the production of the negative 1385-MeV $\Lambda\pi$ resonance and the negative 1535-MeV $\Xi\pi$ resonance and their antiparticles; with some luck on the relative magnitudes of the latter two cross sections, the lower limit on the cross section for $Z\bar{Z}$ production will be nontrivial. As before, of course, the relations (7) hold for either differential or total cross sections, provided only that the mass differences between the various final states may be neglected.

If one has observed all three cross sections involved in (7), then one may determine both the magnitudes and the relative phase of the $U=0$ and $U=1$ production amplitudes in terms of the cross sections. The cross section for the fourth reaction, the production of N^{*-} together with its antiparticle, is thus completely determined so that one obtains an equality rather than an inequality. This turns out to be

$$\sigma_N+3\sigma_\Xi=3\sigma_Y+\sigma_Z, \quad (8)$$

where σ_N is the cross section for the production of N^{*-}–$\bar{N}^{*+}$ pairs. This equality is, unfortunately, not very useful from experimental considerations, as the nucleon isobars can decay only into neutrons and antineutrons, which makes σ_N extremely difficult to measure. To avoid the experimental difficulties, one can use isospin invariance to express $(N^{*-}\bar{N}^{*+})$ in terms of $(N^{*0}\bar{N}^{*0})$ and $(N^{*++}\bar{N}^{*--})$, since the N^{*0} and N^{*++} both have doubly charged decay modes. One thus obtains, for example, the relation

$$-\tfrac{3}{2}(N^{*0}\bar{N}^{*0})+\tfrac{1}{2}(N^{*++}\bar{N}^{*--}) = -2(Y_1^{*-}\bar{Y}_1^{*+})-(\Xi^{*-}\bar{\Xi}^{*+}). \quad (9)$$

Unfortunately, if one wishes to deal only with the N^{*0} and N^{*+} for experimental reasons, a new unknown interference term is introduced so that (9) once again yields only inequalities rather than equalities.

Equation (6), together with Eq. (8) [or the equivalent Eq. (9)], are two of the three restrictions imposed by unitary symmetry. The third restriction is obtained by noting, again from Table I, that the production of N^{*0}, Y_1^{*0}, and Ξ^{*0} with their respective antiparticles is governed by only two amplitudes, again

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[8] I am indebted to J. Gibbs for a particularly convenient way of carrying out the reduction of the direct product by means of ladder operators in a manner similar to the standard treatment of angular momentum.

for $U=0$ and 1. This gives the relation

$$(Y_1^{*0}\bar{Y}_1^{*0})=\frac{1}{2}(\Xi^{*0}\bar{\Xi}^{*0})+\frac{1}{2}(N^{*0}\bar{N}^{*0}) \tag{10}$$

together with the associated triangle inequalities on the cross sections. Y_1^{*0} is once again a difficult particle to observe, since it will normally decay by π^0 emission, so that the inequalities implied by (10) will be difficult to check experimentally. It would again be possible to use isotopic spin invariance to replace $(Y_1^{*0}\bar{Y}_1^{*0})$ by the amplitudes for charged Y_1^* pair production in (10), but only at the cost of weakening the inequalities.

To sum up, the most meaningful and at the same time experimentally accessible predictions of the unitary symmetry theory for Reaction (1) would seem to be the three inequalities (7).

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Absorption of Σ^- Hyperons in Photographic Emulsion Nuclei

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396 Σ^- capture stars in nuclear emulsion have been analyzed. The new data have been combined with 231 similar events previously reported. The rate of hyperfragment emission from Σ^- stars is $(2.7^{+0.5}{}_{-0.4})\%$. Both this rate and an estimate of the frequency of cryptogragment formation are compared with the analogous quantities pertinent to K^- absorptions at rest. Evidence is presented that hyperfragments predominantly originate from Σ^- absorptions in the light elements of the emulsion, while cryptogragments are formed following Σ^- absorption in heavy nuclei.

INTRODUCTION

BECAUSE of the strong Pauli suppression in complex nuclei of the charge-exchange reaction, the absorption at rest of a Σ^- hyperon in an emulsion nucleus is always-expected to create a Λ^0 hyperon with a resultant energy release of about 80 MeV. Sacton *et al.*[1] in a previous work studied the trapping probabilities of these Λ^0 hyperons and found them to be appreciably smaller in both the heavy (silver and bromine) and the light (carbon, nitrogen, and oxygen) emulsion nuclei than those of Λ^0 hyperons produced by K^--meson interactions at rest. The main purpose of this work has been to investigate further the above result by adopting the same procedures as used by Sacton *et al.*[1] but with a much larger sample of events. In addition, however, the chance that a Σ^- capture in silver or bromine gives rise to cryptofragment formation has been estimated not only from the frequency of emission of a fast proton from the capture star but also from a comparison of the visible energy releases in Σ^- captures and the nonmesonic decays of heavy hypernuclei.[2]

EXPERIMENTAL PROCEDURE

Two stacks of 1200 μ-Ilford K5 emulsions, each exposed to stopping K^- mesons at the Berkeley Bevatron and previously used for hyperfragment studies,[3,4] were used in this experiment. The Σ^- capture events were found by following out the grey and black tracks from about 50 000 K^- capture stars, in one stack to their end points in the emulsion, in the other only within the pellicle containing the parent star. Σ^- hyperons of ranges less than 200 μ were rejected from the sample in order to reduce to negligible proportions the background of nonmesonically decaying hyperfragments and also to enable a satisfactory distinction to be made between the tracks of Σ^- hyperons and slow π^- mesons.

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